3.1 Introduction:
If the fluid properties such as density, velocity, depth, pressure at any point in a flowing changes w.r.t time then such flow is referred as unsteady flow.
For unsteady flow $\frac{d}{dt}(\text{flow characteristics}) \neq 0$.
Example-
When a fluid flowing in pipe is suddenly bought to rest its property gets changed resulting unsteady flow.

3.2 Celerity/Propagation speed/Pressure transients/Hydraulic transients “c”:
When the fluid flowing in a pipe is suddenly bought to rest by closing a valve in pipeline, the whole of fluid does not respond to the valve closer simultaneously, instead a pressure discontinuity in the form of wave propagates back the pipeline. The velocity of this pressure wave is called celerity. Celerity is equal to the velocity of sound wave in water.

3.3 Time history of Hydraulic transients:
Development of celerity in pipe is studied in different instances of time which is schematically shown in the following figure.
Consider flow occurring in a pipe of length L which is bought to rest by closing a valve, let c be the speed of pressure wave produced,

![Diagram](image-url)

Stage 1 : When $0<T<L/c$
At this instant of time the valve is just closed, this creates pressure rise which starts at the valve section and gets subsequently propagated throughout the pipe length in opposite direction of flow.

Stage 2: When $T=L/c$
At this moment, pressure wave reaches to reservoir thus pressure rise occurs along the whole length of pipe. There is unbalanced pressure between the reservoir and the pipeline.
Stage 3: $L/c < T < 2L/c$
Pressure inside the pipe is more than that in the reservoir because of which flow starts to occur from pipe to the reservoir and the pressure wave propagates back toward valve.

Stage 4: $T = 2L/c$
All the fluid from pipe moves toward the reservoir and the pressure at both sides is completely equal.

Stage 5: $2L/c < T < 3L/c$
When time is greater than $2L/c$ whole of the fluid from the pipe gets moved to the reservoir due to which there remains no more fluid at the valve section in the pipe so negative pressure travels toward reservoir.

Stage 6: $T = 3L/c$
Since all the water has entered to the reservoir now there is negative pressure throughout the length of the pipe.

Stage 7: $3L/c < T < 4L/c$
At this instant pressure along the pipe is less than that in the reservoir so flow starts to occur from reservoir to pipe till the pressure becomes equal at the both sides.
Stage 8: When \( T = \frac{4L}{c} \)
Pipe gets filled with fluid flowing from reservoir, restoring the initial condition in the pipe.

### 3.4 Propagation of Hydraulic transient: (Derivation of celerity in rigid pipe)
(Rigid pipe, fluid is compressible)
Consider flow occurring in a rigid pipe of length \( L \) which is brought to rest by closing a valve, let \( c \) be the speed of pressure wave produced, assume \( \rho \) be the density of the flowing fluid and \( K \) be its bulk’s modulus of elasticity.

By continuity theorem, mass flux across the pipe is equal at every point

\[
\rho (c + v)A = (\rho + d\rho) c A
\]

Dividing both sides by \( \rho c A \)

\[
(1 + \frac{v}{c} = \frac{d\rho}{\rho} + 1)
\]

\[
\frac{v}{c} = \frac{d\rho}{\rho} \quad \text{Equation i}
\]

By momentum theorem,

Force = Mass flux * change in velocity

\[
P A - (P + dP) A = \rho (c + v) A (c - (c + v))
\]

\[
dP = \rho cu \quad \text{Equation ii}
\]

Elastic property of fluid is determined by its bulk modulus of elasticity,

\[
K = \frac{dP}{(\frac{dv}{v})}
\]

Since \( v \propto \frac{1}{\rho} \)

\[
\frac{dv}{v} = -\frac{d\rho}{\rho}
\]
Thus \( dp = k \frac{dp}{\rho} \) \( \ldots \ldots \ldots \ldots \ldots \). Equation iii

From equation i, ii, iii

\[ c = \sqrt{\frac{k}{\rho}} \]

3.5 Water hammer effects in pipe:

When the water flowing in a long pipe is suddenly bought to rest by closing the valve or by any similar cause then there will be sudden rise in the pressure due to momentum of water being destroyed this causes a wave of high pressure to be transmitted along the pipe creating a knocking noise. This phenomenon of sudden rise in pressure due to closing of valve or due to any other similar cause and subsequent propagation of pressure wave along the pipe causing noise is known as water hammer effect.

Sometime the rise of pressure may get large enough to burst the pipe so pressure needs to be considered in the design of pipe system.

Magnitude of pressure rise depends on

-speed at which valve is closed
-velocity of flow of water in the pipe
-length of the pipe
-elastic properties of pipe material

3.6 Sudden and Gradual closure of valve

If \( T \) be the time required by the pressure wave to travel once up and down the pipe and \( c \) be the celerity then \( T = \frac{2L}{c} \)

Let ‘\( t \)’ be the actual time taken for closer of valve

If “\( t > T \)” then such closure is referred as \textbf{gradual closure} of valve.

When valve closure is gradual fluid is considered to be incompressible and pipe material is considered rigid.

If “\( t < T \)” then such closure is referred as \textbf{instantaneous or sudden closure} of valve.

When valve closure is instantaneous or sudden fluid is considered to be compressible and pipe material is considered both as rigid and elastic.
3.7 Pressure rise due to gradual closer of valve:

Consider flow occurring in a rigid pipe of length L which is brought to rest by gradually closing a valve at time t, let c be the speed of pressure wave produced, assume ρ be the density of the flowing fluid and since the valve closure is gradual fluid is considered to be incompressible.

Retardation due to valve closure = V/t
This produces retardation force along the whole length of the pipe
\[ F = \rho AL \cdot \frac{V}{t} \]
\[ F \cdot A = \frac{\rho LV}{t} \]
\[ P = \frac{\rho LV}{t} \]
\[ P = \frac{\rho g VL}{gt} \]
This gives rise in pressure due to gradual valve closure.

3.8 Pressure rise due to sudden closer of valve:

(a) Pipe material is considered rigid and fluid is compressible

Consider flow occurring in a rigid pipe of length L, cross sectional area A which is brought to rest by suddenly closing a valve at time t, let c be the speed of pressure wave produced, assume ρ be the density of the flowing fluid and K be the bulk modulus of elasticity of fluid.

When fluid is brought to rest there is lost in kinetic energy of the fluid this lost in kinetic energy is converted to gain in pressure energy, since fluid is compressible this pressure energy causes rise in internal energy of the fluid material.

Thus by conservation of energy

Loss in kinetic energy of the fluid = Gain in internal energy of the fluid
Loss in kinetic energy of the fluid throughout the pipe when velocity is reduced to 0 from 
\[ V = \frac{1}{2} \rho Alv^2 \]

Gain in internal energy of the fluid = \( \int_{\text{vol}} \left( \frac{\text{stress}^2}{2 \times \text{Elasticity}} \right) dV \)

Gain in internal energy of the fluid throughout the pipe is \( \frac{p^2 AL}{2K} \)

Thus, \( \frac{1}{2} \rho Alv^2 = \frac{p^2 AL}{2K} \)

\[ P = v \rho \frac{\sqrt{k}}{\rho} \]

\[ P = v \rho c \], where \( c \) is the celerity.

(b) Pipe material is considered elastic and fluid is compressible

Consider flow occurring in elastic pipe of length \( L \), diameter \( d \) and young’s modulus of elasticity \( E \), which is brought to rest by suddenly closing a valve at time \( t \), let \( c \) be the speed of pressure wave produced, assume \( \rho \) be the density of the flowing fluid and \( K \) be the bulk modulus of elasticity of fluid.

Since the pipe material is elastic, total loss in kinetic energy due to valve closure is converted to gain in internal energy of fluid due to its compressibility and gain in internal energy of pipe material due to its elasticity.

Thus by conservation of energy,

Loss in kinetic energy of the fluid = Gain in internal energy of the fluid + Gain in internal energy of the pipe material.

Loss in kinetic energy of the fluid throughout the pipe when velocity is reduced to 0 from 
\[ V = \frac{1}{2} \rho Alv^2 \]
Gain in internal energy of the fluid = \( \int_{vol} \left( \frac{\text{stress}^2}{2 \times \text{Elasticity}} \right) dV \)

Gain in internal energy of the fluid throughout the pipe is \( \frac{P^2AL}{2K} \)

Gain in internal energy of the pipe material per unit volume \( \frac{E_s}{V} = \frac{1}{2E} \left( f_l^2 + f_c^2 - \frac{2f_l f_c}{m} \right) \)

Where \( f_l \) & \( f_c \) is the longitudinal and circumferential stress developed due to expansion of the walls of the pipe, \((1/m)\) is the poisson’s ratio. Putting Poisson’s ratio as 0.25 or \( m = 4 \),

Volume of the pipe material \( V = \pi dt \)

Total gain in strain energy of the pipe material \( E_s = \frac{V}{2E} \left[ \frac{P^2d^2}{16t^2} + \frac{P^2d^2}{4t^2} - \frac{P^2d^2}{16t^2} \right] \)

\( E_s = \frac{\pi dt}{2E} \left[ \frac{P^2d^2}{16t^2} + \frac{P^2d^2}{4t^2} - \frac{P^2d^2}{16t^2} \right] \)

\( E_s = \frac{P^2Adl}{8Et^2} \)

\( E_s = \frac{P^2Adl}{2Et} \)

The energy balance equation is now,

\( \frac{1}{2} \rho ALv^2 = \frac{P^2AL}{2K} + \frac{P^2Adl}{2Et} \)

\( P = \sqrt{\frac{\rho}{\left( \frac{1}{K} + \frac{d}{Et} \right)}} \)

This gives the value of pressure growth in elastic pipe due to sudden closure of valve.

3.9 Relieving devices against water hammer effect:
Water hammer effect or hydraulic transient have severe effect at the pump station, the propagation of high pressure wave may burst the pipe line system and destroy other pipeline equipments. Relieving devices has to be installed to eliminate this effect.

Example of Relieving devices in hydropower plant – Surge tank is an example of one such device that can be installed to reduce water hammer effect.

- Surge tank are provided at the head of penstock pipe and end of tail race tunnel.
- It prevents high pressure rise by receiving water from the pipeline helping to reduce water hammer effect.
- Installation of surge tank leads to economic design of pipeline system.

![Surge tank diagram](image)

Figure: Surge tank

The above figure is an example of surge tank installation in hydropower plant, three distinct HGL (Hydraulic gradient line) is plotted for three different operating conditions.

- HGL under steady load indicates normal HGL when the turbine is operating under uniform flow.
- Increased level of HGL when valve is closed this is the operational situation when the load on turbine is reduced and there is sudden rise in the pressure of penstock pipe.
- Decreased HGL when the valve is reopened this indicates the operational situation when the load on the turbine needs to be increased, the increased demand of water is temporarily taken from the surge tank.

**Types of surge tank**
Closed Cylindrical tank

Open cylindrical tank

Differential surge tank

Restricted orifice surge tank

Hydraulics
Chapter-3
Unsteady flow
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